DOCUMENT RESUME

ED 450 148 TM 032 338

AUTHOR Cool, Angela L.
TITLE A Primer on Regression and Canonical Commonality Analyses: Partitioning Predicted Variance into Constituent Parts.
PUB DATE 2001-02-00
NOTE 26p.; Paper presented at the Annual Meeting of the Southwest Educational Research Association (New Orleans, LA, February 1-3, 2001).
PUB TYPE Reports - Descriptive (141) -- Speeches/Meeting Papers (150)
EDRS PRICE MF01/PC02 Plus Postage.
DESCRIPTORS Correlation; Predictor Variables; *Regression (Statistics)
IDENTIFIERS *Commonality Analysis

ABSTRACT

Multiple regression analysis is used with considerable frequency by researchers as a means of predicting the impact of predictor variables on a dependent variable. Regression predictors are typically correlated, often intentionally. To better understand the relative contribution of each independent variable in regression (and other) analyses, researchers can partition the squared multiple correlation (R squared) into constituent portions that can be attributed to the independent variables both uniquely and in various combinations with each other. The purpose of this paper is to explain and illustrate the use of this "commonality analysis." A small heuristic data set is used to outline the steps in this approach. An appendix contains R squared results for four predictors of depression from the example. (Contains 4 tables and 27 references.) (Author/SLD)

Reproductions supplied by EDRS are the best that can be made
from the original document.

Running head: COMMONALITY ANALYSES

ED 450 148

A Primer on Regression and Canonical Commonality Analyses: Partitioning Predicted Variance into Constituent Parts

Angela L. Cool

Texas A&M University 77843-4225

TM032338

A. L. Cool

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)
☑ This document has been reproduced as received from the person or organization originating it.
☐ Minor changes have been made to improve reproduction quality.
• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

Paper presented at the annual meeting of the Southwest Educational Research Association, New Orleans, February 1-3, 2001.

 BEST COPY AVAILABLE

Abstract

Multiple regression analysis is used with considerable frequency by researchers as a means of predicting the impact of predictor variables on a dependent variable. Regression predictors are typically correlated, often intentionally. To better understand the relative contribution of each independent variable in regression (and other) analyses, researchers can partition the squared multiple correlation (R^2) into constituent portions that can be attributed to the independent variables both uniquely and in various combinations with each other. The purpose of the present paper is to explain and illustrate the use of "commonality analysis." A small heuristic data set is used for this purpose.

A Primer on Regression and Canonical Analyses: Partitioning Predicted Variance into Constituent Parts

Multiple regression analysis is used with considerable frequency by researchers as a means of predicting the impact of predictor variables on a dependent variable (Elmore & Woehlke, 1988; Goodwin & Goodwin, 1985; Willson, 1982). This usage has increased as more researchers have become cognizant that all parametric statistical analyses are part of a single general linear model, e.g., regression, canonical correlation analysis, and structural equation modeling, each in turn (Bagozzi, Fornell, & Larcker, 1981; Fan, 1997).

Regression predictors are typically correlated, often intentionally (Pedhazur, 1982). Notwithstanding myths to the contrary, such collinearity is in no way problematic, except that in such cases both beta weights and structure coefficients must be interpreted (Thompson, 1997; Thompson & Borrello, 1985).

To better understand the relative contribution of each independent variable in regression (and other) analyses, researchers can partition the squared multiple correlation (R^2) into constituent portions that can be attributed to the independent variables both uniquely and in various combinations with each other. This method, "commonality analysis," is useful precisely because it does not depend on *a priori* knowledge of the influence of the predictors. According to Cooley and Lohnes

(1976), "such neutrality allows the information inherent in the data about the value of organizing observations in a certain framework (that of the domains of predictors) to emerge" (p. 219).

Because commonality analysis considers all possible orders of entry of the predictors into the model, there is no distortion of the results such as that which occurs when stepwise analyses are conducted (Snyder, 1991; Thompson, 1995, 1998). As Thompson (1995) explained in detail, stepwise regression analysis does not identify the best predictor set of a certain size. Indeed, the best predictor set of a certain size (a) may have a higher R^2 than the variable set identified by stepwise, and (b) may even include none of the predictors selected by stepwise (cf. Thompson, 1995).

As Seibold and McPhee (1979) explained, commonality analysis decomposes the squared multiple correlation into the proportion of the explained variance of the dependent variable associated (a) uniquely with each independent variable and with the (b) common effects of each. They also noted that this decomposition of R^2 into its unique and common components is rarely conducted and argued that:

> Advancement of theory and the useful application of research findings depend not only on establishing that a relationship exists among

> predictors and the criterion, but also upon determining the extent to which those independent variables, singly and in all possible combinations, share variance with the dependent variable. Only then can we fully know the relative importance of independent variables with regard to the dependent variable in question. (p. 355)

The purpose of the present paper is to explain and illustrate the use of commonality analysis (Cooley & Lohnes, 1976; Mood, 1969; Seibold & McPhee, 1979; Thompson, 1985; Wisler, 1972). A small heuristic data set will be used for this purpose. First, regression commonality analysis will be explained and illustrated. Then a heuristic application will be presented illustrating generalization of the method to canonical commonality analysis (cf. Crossman, 1996).

The Logic of Commonality Analysis

Commonality analysis is a procedure that was originally developed for use in the regression case (Thompson & Miller, 1985; Thompson,1985) and was extended usefully to the canonical case, thus reinforcing the idea that canonical correlation analysis is the most general linear model of classical parametric statistics (Thompson & Borrello, 1985; Thompson & Miller, 1985). As Thompson and Miller (1985, p. 2) explained,

"the [commonality] analysis indicates how much of the explanatory power of a variable is 'unique' to the variable, and how much of the variable's explanatory power of a variable is 'common' to or also available from one or more other variables." The unique contribution of an independent variable can be defined as the squared semi-partial correlation between the dependent variable and the selected independent variable after all other independent variable components have been partialed out (Wisler, 1969). According to Seibold and McPhee (1978, p. 355), "Commonality analysis thus sheds additional light on the magnitude of an obtained multivariate relationship by identifying the relative importance of all independent variables, findings which can be of theoretical and practical significance."

Regression Commonality Analysis

Regression techniques are often utilized by educational and behavioral researchers, although these methods are most useful when the independent variables are uncorrelated and can be experimentally manipulated (Beaton, 1973; Pedhazur, 1982; Rowell, 1996). However, this seems rarely the case in the social sciences. To ascertain the relative contribution of the independent variables uniquely and in various combinations with each other, researchers may partition the squared multiple correlation (R^2) into constituent portions (Rowell, 1996).

Data from a previous study published by Clay, Anderson, and Dixon (1993) in the September/October issue of the Journal of Counseling and Development and found in Murthy's (1994) treatment of commonality analysis will be employed to illustrate the steps in the process of conducting a commonality analysis in the regression case. The study examined undergraduates' perceptions of stress and anger as it was related to depression. In this study, 247 undergraduates completed questionnaires assessing stressful life events, depression, and anger expression. According to Murthy (1994, p. 7),

> The anger expression instrument yielded three different subscales; anger in (IN) anger out (OUT) and anger control (CONT), while the stressful life events instrument yielded only one score (STS). [Furthermore], the results from this study concluded that anger in and stressful life events were significantly related to depression, and that anger out and anger control were not. Thus, the authors decided to eliminate anger out and anger control from further analyses and just focus on the other two variables (STS & IN). But because of the high degree of correlation between all of these predictor variables, (IN, OUT, CONT, & STS), commonality analysis can be used to

determine the unique and common components of these variables so that a more accurate explanation in predicting depression can be obtained.

In the heuristic example presented here, four independent variables were utilized as predictors of depression: anger in, anger out, anger control, and stressful life events. The first step in conducting a commonality analysis for these four predictors is to obtain the equations necessary for computing the unique and commonality components of a four-predictor model. These equations can be found in Table 1.

INSERT TABLE 1 ABOUT HERE.

All the R^2 values, presented in Appendix A, are then substituted into the 15 equations and results are organized in a summary table that is easy to interpret and allows for a quick inspection of arithmetic. The equations are fairly straightforward algebraic product expansions of the independent variables; however, as the number of predictor variables increases, the calculations necessary to obtain the common contributions get increasingly complex (Rowell, 1996).

In general, the number of possible unique and commonality components can be determined by (2^P-1) where P is the number of independent variables employed in the model. Likewise as Rowell

(1996) and others have noted, the number of unique components equals *P* as well because *P* represents the number of independent factors considered; the number of "common" components can then be obtained by the equation $(2^P-1)-P$, the difference between the total number of components and the number of unique components. For example, with the four predictor variables in the present study, the total possible number of components (both unique and common) are $2^4 - 1 = 15$, with 4 being unique components and 11 being the common components.

To illustrate the commonality process, first and foremost, the squared multiple correlation (R^2) values from a statistical program printout (e.g., SAS) must be applied to the appropriate formulas in Table 1. The calculations can be done easily with a spreadsheet program or calculator. For example:

$$\begin{aligned} U4(\text{anger control}) &= -R^2(123) + R^2(1234) \\ &= -.29347 + .30414 \\ &= .01067 \end{aligned}$$

Hence, the unique explanatory contribution of the predictor variable, anger control, to the proportion of total dependent variable (depression) variance was .01067, or approximately 1%. Likewise, the commonality between stress(1) and anger control(4) can be computed as:

$$C14 = -R^2(23) + R^2(123) + R^2(234) - R^2(1234)$$

$$= -.14669 + .29347 + .15977 - .30414$$

$$= .00242$$

Therefore, the common variance in depression among undergraduate students explained in common by either stressful life events or anger control was .00242 or .2%. One would continue with these computations until all 15 unique and common components were determined.

The last step is to place the values obtained into a commonality analysis table, such as the one presented in Table 2. Once the unique and common components are presented in tabular form, arithmetic checks can be performed. As Rowell (1996, p. 37) noted,

> Row entries are the specific unique and commonality effects of each independent variable. The column totals of each independent variable will equal to the R^2 of the regression model in which that independent variable is the only variable entered into the model. Another check is that the sum of all unique and commonality values for all the variables as a set should equal the R^2 value of the regression model when *all* the independent variables are entered into the model.

INSERT TABLE 2 ABOUT HERE.

The commonality analysis summary table presented in Table 2 indicates that the unique variance contribution of the predictor, stress, is approximately 14% (.14437) and the common variance contribution, the proportion of predictive ability of stress that also exists in one or more of the other predictors, is approximately 5% (.04923). Likewise, anger in has a unique contribution of approximately 10% (.10107) and a common variance of approximately 4% (.03583). Unfortunately, the remaining variables (anger out and anger control) offer a small amount of unique contribution to the variance.

Canonical Commonality Analysis

Canonical correlation analysis (Thompson, 1984, 1991, 2000) is a multivariate analytic method for investigating the relationship between two sets of variables, a set of dependent variables and a set of independent variables, where each set contains two or more variables. Each set of variables (predictor and criterion) represents a latent construct which the researcher is examining. As Crossman (1996, p. 96) explained, "one reason that canonical correlation analysis is such a powerful analytical technique is because it considers all the relationships among all the variables and does not require that variables be converted to nominally-scaled variables, which

discards information or distorts reality." Furthermore, because canonical correlation analysis subsumes multiple regression as a special case, and commonality analysis has proven helpful in interpreting multiple regression results (Thompson & Borrello, 1985), interpretation of canonical results is likewise facilitated with the use of commonality analysis (Thompson, 1988).

The steps of canonical commonality analysis are as follows: (a) perform a canonical correlation analysis, (b) calculate the z-scores and criterion composite scores, also called "variate scores", (c) conduct a multiple regression on the synthetic composite criterion variables (obtain regression equations), and (d) calculate the unique and commonality components.

A hypothetical data set found in Campbell's (1990) paper addressing applications of multivariate commonality analysis will be utilized here to make the discussion more concrete. As Campbell (1990, p. 2) noted:

> The hypothetical data set involved 22 cases or observations. The variables were opinions (on a scale of 1-20) about various events which occurred during the Reagan administration. [Two] variables were designated as predictor variables: LESSFED (less federal aid), and MOREDEF (more defense spending). Two variables were designated

criterion variables: MORESS (more spending on social security) and CATMED (catastrophic medicine insurance coverage).

Table 3 presents the hypothetical data.

INSERT TABLE 3 ABOUT HERE.

The first step in canonical commonality analysis involves performing a canonical correlation analysis to obtain the canonical functions and canonical correlation coefficients (Leister, 1996; Thompson & Miller, 1985). The second step in this analysis is to calculate the canonical variate scores (a.k.a. criterion composite scores) for all participants. Thompson (1991, p. 83) provided an introduction to this method of analysis. To do so, as Thompson and Miller (1985) and Leister (1996) explained, the function coefficients are multiplied by the z-scores on the criterion variables. These products are then summed to create the synthetic criterion composite variables-one for each function yielded by the canonical correlation analysis. In the present example, the computations for the two functions would be:

CRIT1 = (-0.752 x zCATMED) + (1.572 x zMORESS)

CRIT2 = (1.820 x zCATMED) + (-1.187 x zMORESS)

The third step involves calculating the regression equations, using all possible combinations of predictors to

predict the synthetic criterion composite scores. It is important to note that when all the predictors are used simultaneously, the squared correlation coefficient always equals the squared canonical correlation since the two analyses are identical in the full model case (Leister, 1996; Thompson & Miller, 1985).

The final step in a canonical commonality analysis involves partitioning the components into unique and common variance effects. There are two predictor variables in this example, so the reader is referred back to Table 1 for the three ($2^2-1 = 3$) formulas required to calculate the unique and commonality components. For example, the calculations for the unique and commonality components for Function 1 are as follows:

U1 = -R2(2) + R2(12) = -.64018 + .71057 = **.15705**

U2 = -R2(1) + R2(12) = -.55352 + .71057 = **.07039**

C12 = R2(1) + R2(2) - R2(12) = .55352 + .64018 - .71057 = **.48313**

The commonality results for each function are presented in Table 4.

INSERT TABLE 4 ABOUT HERE.

As Leister (1996, p. 5) noted in his analysis of the canonical commonality results:

In this example, it is shown that on the first function, the majority of the explanatory power of predictors is common to both variables. Although 64.0% of the variance can be accounted for by the variable "Moredef" alone on the first function, 48.3% of the total explanatory power is common to both predictors. The unique explanatory power of "Moredef" is therefore only 15.7%. The unique explanatory power of "Lessfed" is only 7.0% (.553-.48313). Also, it can be seen that the second function has virtually no explanatory ability.

Conclusions

Commonality analysis is a straightforward method of partitioning variance when relatively few (less than 5 or 6) independent variables are of interest (Rowell, 1996). There is no statistical significance test for commonalities. However, this is not disadvantageous because commonality analysis is generally conducted after a statistically significant canonical correlation has already been found (Thompson & Miller, 1985). Commonality analysis is attractive because the method honors the relationships among variables in a set and the reality to which the researcher is trying to generalize (Thompson, 1985; Thompson & Miller, 1985). As Mood (1969, p. 480) pointed out, "The

independent variables in any social process, and certainly in education, are highly correlated among themselves, and this kind of partition provides measures of the extent to which they overlap each other in their association with the dependent variable."

References

Arnold, M.E., & Thompson, B. (1996). Love style perceptions in relation to personality function. Journal of Social Behavior and Personality, 11, 425-438.

Bagozzi, R. P., Fornell, C., & Larcker, D. F. (1981). Canonical correlation analysis as a special case of structural relations model. Multivariate Behavioral Research, 16, 437-454.

Beaton, A. E. (1973). Commonality. Princeton, NJ: Educational Testing Service. (ERIC Document Reproduction Service No. ED 111 829)

Campbell, K. (1990, January). Methods of multivariate commonality analysis. Paper presented at the annual meeting of the Southwest Educational Research Association, Austin. (ERIC Document Reproduction Service No. ED 315 458)

Cooley, W. W., & Lohnes, P. R. (1976). Evaluation research in Education. New York: Irvington.

Creager, J. A. (1971). Orthogonal and nonorthogonal methods of partitioning regression variance. American Educational Research Journal, 8, 671-676.

Crossman, L. L. (1996). Cross-validation analysis for the canonical case. In B. Thompson (Ed.), Advances in social science methodology (Vol. 4, pp. 95-106). Greenwich, CT: JAI Press.

Elmore, P., & Woehlke, P. (1988). Statistical methods employed in American Educational Research Journal, Educational

Researcher, and Review of Educational Research from 1978 to 1987. Educational Researcher, 17(9), 19-20.

Fan, X. (1997). Canonical correlation analysis and structural equation modeling: What do they have in common? Structural Equation Modeling, 4, 65-79.

Goodwin, L. D., & Goodwin, W. L. (1985). Statistical techniques in AERJ articles, 1979-1983: The preparation of graduate students to read the educational research literature. Educational Researcher, 14(2), 5-11.

Mood, A. R. (1969). Macro-analysis of the American educational system. Operations Research, 17, 770-784.

Murthy, K. (1994, April). Commonality analysis for the regression case. Paper presented at the annual meeting of the American Educational Research Association, New Orleans. (ERIC Document Reproduction Service No. ED 367 694)

Pedhazur, E. J. (1982). Multiple regression in behavioral research: Explanation and prediction (2nd ed.). New York: Holt, Rinehart, and Winston.

Rowell, R. K. (1996). Partitioning predicted variance into constituent parts: How to conduct commonality analysis. In B. Thompson (Ed.), Advances in social science methodology (Vol. 4, pp. 33-44). Greenwich, CT: JAI Press.

Seibold, D. R., & McPhee, R. D. (1979). Commonality analysis: A method for decomposing explained variance in

multiple regression analysis. Human Communication Research, 5, 355-365.

Snyder, P. (1991). Three reasons why stepwise regression methods should not be used by researchers. In B. Thompson (Ed.), Advances in educational research: Substantive findings, methodological developments (Vol. 1, pp. 99-106). Greenwich, CT: JAI Press.

Thompson, B. (1985). Alternate methods for analyzing data from education experiments. Journal of Experimental Education, 54, 50-55.

Thompson, B. (1995). Stepwise regression and stepwise discriminant analysis need not apply here: A guidelines editorial. Educational and Psychological Measurement, 55, 525-534.

Thompson, B. (1991). A primer on the logic and use of canonical correlation analysis. Measurement and Evaluation in Counseling and Development, 24, 80-95.

Thompson, B. (1997). The importance of structure coefficients in structural equation modeling confirmatory factor analysis. Educational and Psychological Measurement, 57, 5-19.

Thompson, B. (1998, April). Five methodology errors in educational research: The pantheon of statistical significance and other faux pas. Invited address presented at the annual

meeting of the American Educational Research Association, San Diego. (ERIC Document Reproduction Service No. ED 419 023)

Thompson, B. (2000). Canonical correlation analysis. In L. Grimm & P. Yarnold (Eds.), Reading and understanding more multivariate statistics (pp. 285-316). Washington, DC: American Psychological Association.

Thompson, B., & Borrello, G. M. (1985). The importance of structure coefficients in regression research. Educational and Psychological Measurement, 45, 203-209.

Thompson, B., & Miller, J. H. (1985, February). A multivariate method of commonality analysis. Paper presented at the annual meeting of the Southwest Educational Research Association, Austin. (ERIC Document Reproduction Service No. ED 263 151)

Willson, V. (1982, January). Misuses of regression approaches to ANOVA and ANCOVA. Paper presented at the annual meeting of the Southwestern Educational Research Association, Austin, TX. (ERIC Document Reproduction Service No. ED 222 522)

Wisler, C. E. (1969). Partitioning the variance explained in a regression analysis. In G. Mayeske, C. Wisler, A. Beaton, Jr., F. Weinfield, W. Cohen, T. Okada, J. Proshek, & K. Tabler, A study of our nation's schools. Washington, DC: U.S. Government Printing Office.

Table 1

Formulas for Unique and Commonality Components of Variance

Two Independent Variables

$U1 = -R^2(2)+R^2(12)$
$U2 = -R^2(1)+R^2(12)$

$C12 = R^2(1)+R^2(2)-R^2(12)$

Three Independent Variables

$U1 = -R^2(23)+R^2(123)$
$U2 = -R^2(13)+R^2(123)$
$U3 = -R^2(12)+R^2(123)$

$C12 = -R^2(3)+R^2(13)+R^2(23)-R^2(123)$
$C13 = -R^2(2)+R^2(12)+R^2(23)-R^2(123)$
$C23 = -R^2(1)+R^2(12)+R^2(13)-R^2(123)$
$C123 = R^2(1)+R^2(2)+R^2(3)-R^2(12)+R^2(13)-R^2(23)+R^2(123)$

Four Independent Variables

$U1 = -R^2(234)+R^2(1234)$
$U2 = -R^2(134)+R^2(1234)$
$U3 = -R^2(124)+R^2(1234)$
$U4 = -R^2(123)+R^2(1234)$

$C12 = -R^2(34)+R^2(134)+R^2(234)-R^2(1234)$
$C13 = -R^2(24)+R^2(124)+R^2(234)-R^2(1234)$
$C14 = -R^2(23)+R^2(123)+R^2(234)-R^2(1234)$
$C23 = -R^2(14)+R^2(124)+R^2(134)-R^2(1234)$
$C24 = -R^2(13)+R^2(123)+R^2(134)-R^2(1234)$
$C34 = -R^2(12)+R^2(123)+R^2(124)-R^2(1234)$

$C123 = -R^2(4)+R^2(14)+R^2(24)+R^2(34)-R^2(124)-R^2(134)-R^2(234)+R^2(1234)$
$C124 = -R^2(3)+R^2(13)+R^2(23)+R^2(34)-R^2(123)-R^2(134)-R^2(234)+R^2(1234)$
$C134 = -R^2(2)+R^2(12)+R^2(23)+R^2(24)-R^2(123)-R^2(124)-R^2(234)+R^2(1234)$
$C234 = -R^2(1)+R^2(12)+R^2(13)+R^2(14)-R^2(123)-R^2(124)-R^2(134)+R^2(1234)$
$C1234 = R^2(1)+R^2(2)+R^2(3)+R^2(4)+R^2(12)+R^2(13)-R^2(14)-R^2(23)-R^2(24)$
$-R^2(34)+R^2(123)+R^2(124)+R^2(134)+R^2(234)-R^2(1234)$

Note. From "Partitioning predicted variance components into constituent parts: How to conduct regression commonality analysis," by K. Rowell, 1996.

Table 2

Regression Commonality Analysis Summary Table

Component	1 STRESS	2 ANGER IN	3 ANGER OUT	4 ANGER CONTROL
U1	.14437			
U2		.10107		
U3			.00016	
U4				.01067
C12	.03917	.03917		
C13	.00241		.00241	
C14	.00242			.00242
C23		.00052	.00052	
C24		-.00452		-.00452
C34			.0027	.0027
C123	.00203	.00203	.00203	
C124	-.00113	-.00113		-.00113
C134	.00451		.00451	.00451
C234		-.00006	-.00006	-.00006
C1234	-.00018	-.00018	-.00018	-.00018
UNIQUE	.14437	.10107	.00016	.01067
COMMON	.04923	.03583	.01194	.00373
TOTAL	.1.36	.13690	.01210	.01440

Note. From "Commonality analysis for the regression case," by K. Murthy, 1994.

Table 3

Original Hypothetical Data

	Criterion Variables		Predictors	
OBS	CATMED	MORESS	LESSFED	MOREDEF
1	15	16	20	20
2	14	14	19	19
3	12	13	10	11
4	14	13	9	10
5	15	15	8	9
6	15	14	7	8
7	17	16	20	19
8	13	15	19	19
9	15	16	18	19
10	14	16	17	17
11	10	12	15	15
12	10	11	8	8
13	10	9	8	6
14	14	15	18	17
15	13	13	10	10
16	15	15	17	17
17	16	16	20	19
18	14	15	19	20
19	14	14	16	16
20	13	13	10	9
21	11	12	15	15
22	13	12	9	9

Note. Adapted from "Canonical commonality analysis," by K. D. Leister, 1996.

Table 4

Canonical Commonality Analysis Summary Table

Component	1 LESSFED	2 MOREDEF
Function 1		
U1	.07039	
U2		.15705
C12	.48313	.48313
TOTAL	.55352	.64018
Function 2		
U1	.00031	
U2		.00027
C12	-.00024	-.00024
TOTAL	.00007	.00003

Note. From "Canonical commonality analysis," by K. D. Leister, 1996.

Appendix A

R-Square Results for Four Predictors of Depression

Number of Predictors	R-Square	Variables in Model
1	.19360	1 STRESS
1	.13690	2 ANGER IN
1	.01210	3 ANGER OUT
1	.01440	4 ANG CONTROL
2	.29061	1 2
2	.20239	1 4
2	.19692	1 3
2	.15719	2 4
2	.14669	2 3
2	.01953	3 4
3	.30398	1 2 4
3	.29347	1 2 3
3	.20307	1 3 4
3	.15977	2 3 4
4	.30414	1 2 3 4

Note. From "Commonality analysis for the regression case," by K. Murthy, 1994.